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Janus solutions in M-theory[1]

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Abstract: We present a one-parameter deformation of the $AdS_4 \times S^7$ vacuum, which is a regular solution in M-theory, invariant under $\mathrm{SO}(2,2)\times\mathrm{SO}(4)\times\mathrm{SO}(4)$, and which preserves 16 supersymmetries. The solution corresponds to a holographic realization of a Janus-like interface/defect theory, despite the absence of a dilaton in M-theory. The 2+1-dimensional CFT dual results from the maximally symmetric CFT through the insertion of a dimension 2 operator which is localized along a 1+1-dimensional linear interface/defect, thereby partially breaking the superconformal symmetry. The solution admits a regular ABJM reduction to a quotient solution which is invariant under $\mathrm{SO}(2,2)\times\mathrm{SO}(4)\times\mathrm{U}(1)^2$, preserves 12 supersymmetries, and provides a Janus-like interface/defect solution in ABJM theory.

Keywords: AdS-CFT Correspondence, M-Theory

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Contents

1 Introduction

The Janus solution in Type IIB supergravity [1] provides one of the simplest deformations of the maximally supersymmetric $AdS_5 \times S^5$ solution. The original Janus solution has vanishing 3-form fields and breaks all supersymmetries, but solutions with non-vanishing 3-form fields which preserve various degrees of supersymmetry exist as well. Janus solutions with 4 and 16 supersymmetries were constructed respectively in [2, 3], and in [4, 5][1]. The Type IIB Janus solutions exhibit a common characteristic, namely their non-trivial space-dependence of the dilaton field, and smooth interpolation between several different asymptotic $AdS_5 \times S^5$ regions, each of which has an independent constant dilaton expectation value. The holographic duals to these Janus solutions are interface/defect theories in which the gauge coupling is constant throughout the bulk of each half-space, but is allowed to jump across a planar 2+1-dimensional interface/defect, where the half-spaces join together. Generally, local gauge invariant operators which are localized on the interface/defect may be inserted, and these are in fact required for supersymmetry [10, 11].

At first sight, it may seem that no Janus solutions can exist in M-theory, since there is no dilaton field in M-theory.

[1] See [6–9] for earlier and related work on the holographic description of BPS defects.

In this paper we will show that, contrary to this naive expectation, there actually exists a regular one-parameter family of deformations of the maximally symmetric $AdS_4 \times S^7$ solution for M-theory, which is invariant under $\mathrm{SO}(2,2) \times \mathrm{SO}(4) \times \mathrm{SO}(4)$, preserves 16 supersymmetries, and can be interpreted as a Janus-like solution in M-theory. Its full supergroup invariance is $\mathrm{OSp}(4|2,\mathbf{R}) \times \mathrm{OSp}(4|2,\mathbf{R})$. This family of solutions is characterized not by the space-dependence of the dilaton — since there is no dilaton in M-theory – but rather by the space-dependence of the 4-form field of M-theory. Holographically, these fields correspond to adding counterterms which are localized at the interface/defect. As we shall show below, the Janus-like deformation in M-theory corresponds to the addition of such counterterms for the M2-brane CFT.

The remainder of this paper is organized as follows. In section 2, we begin by carrying out a linearized analysis in an AdS_4 background for the effect of inserting a dimension 2 operator on the interface/defect. We shall confirm that smooth deformations exist at this order. In section 3, we exhibit the exact one-parameter family of Janus-like solutions in M-theory, and discuss their holographic interpretation. In section 4, we show that our Janus solutions are invariant under the Z_k transformations required to carry out the Aharony, Bergman, Jafferis, Maldacena (ABJM) [12] reduction of $AdS_4 \times S^7$ to $AdS_4 \times CP_3$ in Type IIA. Quotienting our Janus deformations of $AdS_4 \times S^7$ by Z_k, $k \neq 1, 2$, yields new solutions which are invariant under $\mathrm{SO}(2,2) \times \mathrm{SO}(4) \times \mathrm{U}(1)^2$, preserve 12 supersymmetries, and exhibit invariance under the supergroup $\mathrm{OSp}(3|2,\mathbf{R}) \times \mathrm{OSp}(3|2,\mathbf{R})$.

2 Linearized analysis

The starting point of the linearized analysis is 4-dimensional gravity with negative cosmological constant, minimally coupled to a scalar or pseudo-scalar field ϕ of mass m. In the following we recall some basic features of the AdS/CFT correspondence [13–15] (for reviews, see [16, 17]). The action is given by,

$$S = \int d^4x \sqrt{-g} \Big(R - \Lambda - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 \Big) \tag{2.1}$$

The Janus solution [1] provides a simple holographic description of an interface conformal field theory. Following this example, we parametrize the AdS_4 metric by a slicing coordinate μ, and a transverse AdS_3 space for each value of μ, and choose ϕ to only depend on μ,

$$ds^2 = f(\mu) \Big(d\mu^2 + ds^2_{AdS_3} \Big) \qquad\qquad \phi = \phi(\mu) \tag{2.2}$$

We set the cosmological constant equal to $\Lambda = -6$, so that the AdS_4 space has unit radius. The pure AdS_4 solution is then given by,

$$f(\mu) = \frac{1}{\cos^2(\mu)} \qquad\qquad \phi = 0 \tag{2.3}$$

where the range of μ is $\mu \in [-\pi/2, \pi/2]$.

To obtain a Janus-like deformation, at linearized order, we choose the mass-square of the field ϕ to be $m^2 = -2$. To justify this choice, we recall the AdS/CFT relation between

m^2 and the conformal dimension Δ of the operator which is dual to ϕ in the dual CFT,

$$m^2 = \Delta(\Delta - 3) \tag{2.4}$$

For $m^2 = -2$, this equation has two solutions, namely $\Delta = 1$, and $\Delta = 2$. As a result, near the boundary components of AdS_4 at $\mu \sim \pm\pi/2$, the asymptotic behavior of ϕ is given by,

$$\phi(\mu) = \phi_1(\mu \mp \pi/2) + \phi_2(\mu \mp \pi/2)^2 + \cdots \tag{2.5}$$

where ϕ_1 and ϕ_2 are constants. Note that the mass $m^2 = -2$ lies in the range $-9/4 < m^2 < -5/4$ where both the modes associated with ϕ_1 and ϕ_2 in (2.5) are normalizable. Hence, there is an ambiguity in identifying which mode sources the operator and which mode turns on the expectation value. This ambiguity is related to the fact that the equation (2.4) for $m^2 = -2$ has two solutions corresponding to operators of conformal dimension $\Delta = 1$ and $\Delta = 2$. Generally, in unitary CFTs, both scaling dimensions are allowed. The ambiguity can be resolved, however, for supergravity theories [18, 19]. For the case at hand, the results of these papers imply that pseudo-scalar fields of mass $m^2 = -2$ correspond to operators with $\Delta = 2$, for which the mode ϕ_1 sources the operator, while ϕ_2 turns on its expectation value. Scalar fields of mass $m^2 = -2$ correspond to operators with $\Delta = 1$, for which the roles of ϕ_1 and ϕ_2 are reversed.

In the remainder of the paper, we will focus on the first case. Thus, ϕ will be a pseudo-scalar, and we shall denote the CFT dual operator by $\mathcal{O}_2$; its dimension is $\Delta = 2$. The operator $\mathcal{O}_2$ would be sourced — in the bulk of the CFT — by the ϕ_1 mode. We shall set $\phi_1 = 0$, so that the operator is *not sourced* in the bulk of the CFT. Thus, in the gravity dual, $\mathcal{O}_2$ will not be sourced in the two boundary half-spaces that are dual to the bulk of the CFT. The mode ϕ_2 then corresponds to the vacuum expectation value of $\mathcal{O}_2$ in the bulk of the CFT. The operator $\mathcal{O}_2$ will be sourced on the interface/defect, however, confirming that its conformal dimension 2 is precisely the one needed to maintain conformal invariance on the interface/defect.

In the following, we consider the linearized problem of small fluctuations around the AdS_4 background (2.3). The equation for the pseudo-scalar field ϕ becomes,

$$\frac{d^2\phi}{d\mu^2} + 2\tan\mu\frac{d\phi}{d\mu} + \frac{2}{\cos^2\mu}\phi = 0 \tag{2.6}$$

which can be solved exactly,

$$\phi(\mu) = \phi_1 \cos\mu \sin\mu + \phi_2 \cos^2\mu \tag{2.7}$$

Upon setting $\phi_1 = 0$, as was advocated above, the linearized solution $\phi(\mu)$ of (2.7) indeed reproduces the asymptotic behavior of (2.5). Note that both the metric back-reaction as well as the coupling of the pseudo-scalar ϕ to other fields will be of order $(\phi_2)^2$ and can be neglected in the linearized approximation. The solution of the full non-linear equation can only be obtained numerically and will not be needed in this paper, since an exact Janus solution of 11-dimensional supergravity will be presented in section 3.

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2.1 Holographic interpretation

In this section we adapt an argument given in [4] to the context of the AdS_4/CFT_3 correspondence in M-theory. Our goal is to confirm that the linearized solution presented in the previous section indeed corresponds to sourcing the operator $\mathcal{O}_2$ dual to the field ϕ on a 1+1-dimensional interface/defect, instead of sourcing $\mathcal{O}_2$ on the entire 2+1-dimensional boundary. The metric for the AdS_3 slicing of AdS_4 takes the following form near $\mu = \pm\pi/2$.

$$\begin{aligned} ds^2 &= \frac{1}{\cos^2\mu}\Big(d\mu^2 + \frac{1}{z^2}(-dt^2 + dx^2 + dz^2)\Big) \\ &= \frac{1}{z^2\cos^2\mu}\Big(z^2 d\mu^2 + (-dt^2 + dx^2 + dz^2)\Big) \end{aligned} \tag{2.8}$$

It is now easy to see that the boundary of AdS_4 in this coordinate system consists of three components [1]. The limits $\mu \to \pm\pi/2$ correspond to two 2+1-dimensional half spaces (since the range of z is the positive real numbers). The two half spaces are glued together along a 1+1-dimensional interface/defect which is associated with the boundary of the AdS_3 slice and is reached as $z \to 0$ with μ arbitrary. With the definition $\epsilon = |\cos(\mu)|z$, the boundary components are reached uniformly as $\epsilon \to 0$. The boundary source for an operator $\mathcal{O}_2$ can be obtained by isolating the term in ϕ which scales like $\epsilon^{3-\Delta}$.

$$\begin{aligned} \phi_{\text{source}} &= \lim_{\epsilon\to 0}\left(\epsilon^{\Delta-3}\phi(\mu)\right) \\ &= \lim_{\epsilon\to 0}\left(\frac{|\cos(\mu)|}{z}\phi_2\right) \end{aligned} \tag{2.9}$$

The limits depends upon the direction along which $\varepsilon = 0$ is being approached. As one keeps $z \neq 0$ fixed, and takes $\mu \to \pm\pi/2$, the two half spaces $\mu = \pm\pi/2$ are approached away from the interface/defect. It follows from (2.9) that this limit, and thus the source for the $\mathcal{O}_2$ operator, vanishes. This means the dual operator $\mathcal{O}_2$ is not inserted in the boundary CFT away from the interface/defect.

The interface/defect is approached as one takes $z \to 0$ with $\mu \neq \pm\pi/2$. In this case the limit in (2.9), and thus the source for the $\mathcal{O}_2$ operator, diverges. This behavior indicates the presence of a Dirac δ-function source for the operator $\mathcal{O}_2$ on the 1+1-dimensional interface/defect. This may be established directly by integrating ϕ_{source} over a small disk around $\epsilon = 0$ in the z, μ-plane. The corresponding integral is given by $\int d\mu\, dz\, z\, \phi_{\text{source}}$ and is finite. Its interpretation is that a term which has δ-function support on the interface/defect is being added to the Lagrangian of the 2+1-dimensional CFT,

$$\mathcal{L} = \mathcal{L}_{CFT_3} + \lambda\, \delta(x^\perp)\mathcal{O}_2 \tag{2.10}$$

where $x^\perp$ is the coordinate transverse to the 1+1-dimensional interface/defect. The linearized analysis corresponds to a small perturbation with $\lambda \ll 1$. Since the conformal dimension of the operator $\mathcal{O}_2$ is two, its addition to $\mathcal{L}_{CFT_3}$ preserves the (global) 1+1-dimensional conformal symmetry of the interface/defect, but breaks the full 2+1-dimensional conformal symmetry of the CFT.

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spin	Dynkin label	m^2	Δ
2	$[n,0,0,0]$	$1/4\,n(n+6)$	$1/2(n+6)$
1	$[n,1,0,0]+[n-1,0,1,1]+[n-2,1,0,0]$	$1/4\,n(n+2)$	$1/2(n+4)$
0^+	$[n+2,0,0,0]+[n-2,2,0,0]+[n-2,0,0,0]$	$1/4(n+2)(n-4)$	$1/2(n+2)$
0^-	$[n,0,2,0]+[n-2,0,0,2]$	$1/4(n(n+2)-8)$	$1/2(n+4)$

Table 1. KK towers of bosonic fields from the supergravity multiplet for $n = 0, 1, 2, \cdots$. Representations with negative Dynkin-labels are to be omitted. Here, m^2 is the mass of the supergravity field, and Δ is the dimension of its dual CFT operator.

spin	SO(8) representation	mass m^2	dimensions Δ
2	$\mathbf{1}$	0	3
1	$\mathbf{28}$	0	2
0^+	$\mathbf{35_v}$	-2	1
0^-	$\mathbf{35_c}$	-2	2

Table 2. The bosonic fields of 4-dimensional $\mathcal{N} = 8$ gauged supergravity with the mass of the fields and the dimension of the dual operator in the CFT.

2.2 $\mathcal{N} = 8$ gauged supergravity

In this subsection, we shall inspect the supergravity fields on $AdS_4 \times S^7$ and identify viable candidates for the pseudo-scalar deformations studied above. The spectrum of the Kaluza-Klein (KK) compactification of M-theory on $AdS_4 \times S^7$ has been obtained by [20, 21].[2] One gets infinite towers of KK-states organized in representations of the SO(8) R-symmetry group, as collected in table 1.

There exists a consistent truncation of the theory to the "massless" $\mathcal{N} = 8$ multiplet which produces $\mathcal{N} = 8$ gauged supergravity in four dimensions and is given by the representations with $n = 0$, summarized in table 2.

Note that the 70 scalars of gauged supergravity split into 35 scalars (denoted 0^+ in table 2) and 35 pseudo-scalars scalars (denoted by 0^-). In the KK-reduction the scalars are obtained from the reduction of the metric component on the sphere whereas the pseudo-scalars are obtained from the AST field strength on the sphere. As discussed in section 2, the ambiguity for the conformal dimensions for the operators dual to the (pseudo-) scalars was resolved in [18, 19] and leads to the values displayed in table 2.

In gauged $\mathcal{N} = 8$ supergravity a Janus interface/defect configuration can be obtained applying the linearized analysis of section 2 for a pseudo-scalar transforming in the $\mathbf{35_c}$ representation of the SO(8) R-symmetry. This field should be dual to a dimension 2 operator in the CFT_3 defined by the decoupling limit of a large number of M2-branes.

In principle one could try to solve the equations of motion for the $\mathcal{N} = 8$ gauged supergravity and a Janus Ansatz to obtain a fully non-linear solution dual to the insertion of such an operator. Solving the full second order equations of motion is, however,

[2] See [22] for a dictionary between supergravity and AdS/CFT conventions.

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prohibitively complicated. A different approach is to ask wether there are interface/defect solution which preserve some of the 32 supersymmetries and correspond to superconformal interface/defects. Solving the resulting BPS-equation is generically easier than solving the equations of motion.

Note that the $\mathbf{35_c}$ representation can be characterized as the rank four self-dual antisymmetric tensor representation of SO(8). Turning on such a field will therefore break the SO(8) R-symmetry down to $\mathrm{SO}(4)\times\mathrm{SO}(4)$. Hence one expects the resulting theory to have $\mathrm{SO}(4)\times\mathrm{SO}(4)$ unbroken R-smmetry.

In the following we will use the results of [23] to obtain a solution of 11-dimensional supergravity, which preserves sixteen supersymmetries, is locally asymptotic to $AdS_4\times S^7$, preserves $\mathrm{SO}(2,2)\times\mathrm{SO}(4)\times\mathrm{SO}(4)$ symmetry and has all the characteristics of a fully non-linear and back-reacted interface/defect solution discussed above.

3 The half-BPS Janus solutions in M-theory

The linearized analysis for a pseudo-scalar field ϕ which is dual to a dimension 2 gauge invariant operator $\mathcal{O}_2$ localized on a 1+1-dimensional interface/defect, and the inspection of multiplets for such fields in gauged $\mathcal{N}=8$ supergravity on AdS_4, reveal a natural candidate for an M-theory Janus solution. The conformal invariance of the interface/defect theory, which is expected to be maintained by the operator $\mathcal{O}_2$, imposes SO(2,2) symmetry. An expectation value to an operator in the $\mathbf{35_c}$ representation of SO(8) leaves a residual $\mathrm{SO}(4)\times\mathrm{SO}(4)$ symmetry, as explained in the preceding section. Thus, the full bosonic symmetry of the corresponding Janus solution in M-theory should be $\mathrm{SO}(2,2)\times\mathrm{SO}(4)\times\mathrm{SO}(4)$. Remarkably, such solutions exist and preserve 16 supersymmetries, a result we shall establish in the present section.

The symmetry and supersymmetry conditions, advocated in the preceding paragraph, place the problem precisely in the context of the general analysis of half-BPS solutions in M-theory with $\mathrm{SO}(2,2)\times\mathrm{SO}(4)\times\mathrm{SO}(4)$ symmetry, for which the general local exact solution was derived in [23] (for space-times asymptotic to either $AdS_4\times S^7$ or $AdS_7\times S^4$). We begin by briefly reviewing the salient points of the solutions obtained in [23]. The 11-dimensional metric Ansatz consists of a fibration of the unit radius metric of $AdS_3\times S_2^3\times S_3^3$ over a 2-dimensional Riemann surface Σ with boundary $\partial\Sigma$,

$$ds^2 = f_1^2 ds^2_{AdS_3} + f_2^2 ds^2_{S_2^3} + f_3^2 ds^2_{S_3^3} + ds^2_\Sigma \tag{3.1}$$

where $ds^2_{AdS_3}$ and $ds^2_{S^3_{2,3}}$ denote the metrics with unit radius on the corresponding spaces, which are invariant respectively under SO(2,2) and SO(4). The $\mathrm{SO}(2,2)\times\mathrm{SO}(4)\times\mathrm{SO}(4)$-invariant Ansatz for the 3-form gauge potential C_3, and for the 4-form field strength $F_4 = dC_3$ are given as follows,

$$\begin{aligned} C_3 &= b_1\hat\omega_{AdS_3} + b_2\hat\omega_{S_2^3} + b_3\hat\omega_{S_3^3} \\ F_4 &= g_{1a}\omega_{AdS_3}\wedge e^a + g_{2a}\,\omega_{S_2^3}\wedge e^a + g_{3a}\,\omega_{S_3^3}\wedge e^a \end{aligned} \tag{3.2}$$

where $\hat\omega_{AdS_3}$ and $\hat\omega_{S^3_{2,3}}$ are the volume forms on the unit-radius spaces AdS_3 and $S^3_{2,3}$ respectively, and e^a, for $a=1,2$ is an orthonormal frame on Σ. The volume forms of the full space-time metric are related to the ones with unit volume by $\omega_{AdS_3} = f_1^3 \,\hat\omega_{AdS_3}$, and $\omega_{S^3_{2,3}} = f_{2,3}^3 \,\hat\omega_{S^3_{2,3}}$. In terms of an arbitrary system of local complex coordinates $w, \bar w$ on Σ, the metric ds^2_Σ in (3.1) reduces to the conformal form,

$$ds^2_\Sigma = 4\rho^2 |dw|^2 \tag{3.3}$$

The metric factors f_1, f_2, f_3, ρ, as well as the flux fields b_1, b_2, b_3, and g_{1a}, g_{2a}, and g_{3a}, only depend on Σ. The Ansatz automatically respects $SO(2,2)\times SO(4)\times SO(4)$ symmetry, which may also be viewed as the symmetry of an AdS/CFT dual 1+1-dimensional conformal interface/defect in the 3-dimensional M2 brane CFT.

3.1 Equations for 1/2 BPS solutions

In [23], the BPS equations governing half-BPS solutions were reduced to constructing a Riemann surface Σ with boundary, a real positive harmonic function h on Σ, and the solution to a first order partial differential equation on Σ for a complex-valued field G, subject to a point-wise quadratic constraint. The partial differential equation for G is given by,

$$\partial_w G = \frac{1}{2}(G + \bar G)\partial_w \ln h \tag{3.4}$$

for an arbitrary complex coordinate system $w, \bar w$ on Σ. For solutions which are locally asymptotic to $AdS_4 \times S^7$ (referred to as "case I" in [23]), the field G is subject to the following point-wise quadratic constraint,

$$|G(w,\bar w)|^2 \geq 1 \qquad\qquad \text{for all } (w,\bar w) \in \Sigma \tag{3.5}$$

We define the following function W on Σ,

$$W^2 \equiv 4|G|^4 + (G - \bar G)^2 \tag{3.6}$$

Assuming that the point-wise quadratic constraint $|G|^2 \geq 1$ is obeyed, we automatically have $W^2 \geq 0$, so that W is real. The special value $W^2 = 0$ corresponds to either $G = +i$ or $G = -i$. The metric factors in (3.1) are then expressed as follows,,

$$\begin{aligned} f_1^6 &= \frac{h^2 W^2}{16^2(|G|^2-1)^2} \\ f_2^6 &= \frac{h^2(|G|^2-1)}{4\,W^4}\left(2|G|^2 + i(G-\bar G)\right)^3 \\ f_3^6 &= \frac{h^2(|G|^2-1)}{4\,W^4}\left(2|G|^2 - i(G-\bar G)\right)^3 \end{aligned} \tag{3.7}$$

The metric factor ρ in (3.3) is given by,

$$\rho^6 = \frac{|\partial_w h|^6}{16^2 h^4}\left(|G|^2-1\right)W^2 \tag{3.8}$$

The anti-symmetric tensor field-strengths are expressed in terms of g_{1a}, g_{2a}, and g_{3a}. They can be simply related to currents $\partial_w b_1$, $\partial_w b_2$, and $\partial_w b_3$, on Σ, which are conserved as a result of the Bianchi identities, and are given as follows,

$$
\begin{aligned}
(f_1)^3 g_{1w} = -\partial_w b_1 &= -\frac{3W^2\partial_w h}{32G(|G|^2-1)} - \frac{1+|G|^2}{16G(|G|^2-1)^2} J_w \\
(f_2)^3 g_{2w} = -\partial_w b_2 &= -\frac{(G+i)\left(2|G|^2+i(G-\bar G)\right)^2}{W^4} J_w \\
(f_3)^3 g_{3w} = -\partial_w b_3 &= +\frac{(G-i)\left(2|G|^2-i(G-\bar G)\right)^2}{W^4} J_w
\end{aligned}
\tag{3.9}
$$

where the following intermediate quantity was used for notational compactness,

$$
J_w = \frac{1}{2}(G\bar G - 3\bar G^2 + 4G\bar G^3)\partial_w h + hG\partial_w \bar G \tag{3.10}
$$

It was shown in [23] that the equations of motion of as well as the Bianchi identities are satisfied for any harmonic h and any G which solves (3.4).

3.2 The $AdS_4 \times S^7$ solution

The simplest solution is the maximally symmetric $AdS_4 \times S^7$ itself. The corresponding Riemann surface is the infinite strip,

$$
\Sigma = \{w \in \mathbf{C},\ w = x+iy,\ x \in \mathbf{R},\ 0 \le y \le \pi/2\} \tag{3.11}
$$

Note that the Riemann surface Σ has two boundary components, namely $y = 0$ and $y = \pi/2$. In these coordinates, the functions h and G for the $AdS_4 \times S^7$ solution are given by,[3]

$$
\begin{aligned}
h &= 4i\big(\mathrm{sh}(2w) - \mathrm{sh}(2\bar w)\big) \\
G &= i\frac{\mathrm{ch}(w+\bar w)}{\mathrm{ch}(2\bar w)}
\end{aligned}
\tag{3.12}
$$

It is easy to check that the partial differential equation (3.4) as well as the point-wise quadratic constraint (3.5) are satisfied. Using (3.7), the metric factors become,

$$
f_1 = \mathrm{ch}(2x) \qquad f_2 = 2\cos(y) \qquad f_3 = 2\sin(y) \qquad\qquad \rho = 1 \tag{3.13}
$$

The boundary may be characterized by the vanishing of the harmonic function $h = 0$, or alternatively, by $G = \pm i$. On the lower boundary of the strip Σ where $y = 0$ one has $G = +i$, which implies that the radius f_2 of S_2^3 vanishes. On the upper boundary of the strip Σ where $y = \pi/2$ one has $G = -i$, which implies that the radius f_3 of S_3^3 vanishes. The boundary of AdS_4, on the other hand, is located at $x = \pm\infty$.

[3] For simplicity, we shall exhibit the solutions with unit AdS_4-radius. The solution for general AdS_4-radius R_0 may be obtained by scaling $h \to R_0^3 h$, while leaving Σ, $w, \bar w$, and G unchanged. As a result, the metric factors scale as follows, $f_i \to R_0 f_i$, $\rho \to R_0\rho$, while the flux fields scale as $b_i \to R_0^3 b_i$ for $i = 1,2,3$.

3.3 The half-BPS Janus solution

It turns out that there is a simple deformation of $AdS_4 \times S^7$ (reviewed in the previous section) which corresponds to a Janus solution in M-theory. As before, the holomorphic coordinate is denoted as $w = x + iy$ and the strip is parametrized as (3.11). The harmonic function h is taken to be proportional to the one of (3.12), but is rescaled in order for the asymptotic $AdS_4 \times S^7$ solution to have the same radius as the undeformed solution (3.12), namely with unit AdS_4-radius,

$$h = \frac{4i}{\sqrt{1+\lambda^2}} \Big(\mathrm{sh}(2w) - \mathrm{sh}(2\bar{w}) \Big) \tag{3.14}$$

The expression for G is given by

$$G = i \frac{\mathrm{ch}(w+\bar{w}) + \lambda\, \mathrm{sh}(w-\bar{w})}{\mathrm{ch}(2\bar{w})} \tag{3.15}$$

It is easy to check that the partial differential equation (3.4) as well as the positivity constraint (3.5) are satisfied for any real value of λ. The solution forms a one-parameter deformation of $AdS_4 \times S^7$, which one recovers by setting $\lambda = 0$.

The metric factors can be expressed concisely in terms of two functions,

$$\begin{aligned} F_+(x,y) &= 1 + 2\lambda \Big(\mathrm{sh}(2x) + \lambda \Big) \cos^2(y) / \mathrm{ch}^2(2x) \\ F_-(x,y) &= 1 - 2\lambda \Big(\mathrm{sh}(2x) - \lambda \Big) \sin^2(y) / \mathrm{ch}^2(2x) \end{aligned} \tag{3.16}$$

The metric factors (3.7) become,

$$\begin{aligned} f_1 &= \frac{\mathrm{ch}(2x)}{\sqrt{1+\lambda^2}} F_+(x,y)^{\frac{1}{6}} F_-(x,y)^{\frac{1}{6}} \\ f_2 &= 2\cos(y)\, F_+(x,y)^{\frac{1}{6}} F_-(x,y)^{-\frac{1}{3}} \\ f_3 &= 2\sin(y)\, F_-(x,y)^{\frac{1}{6}} F_+(x,y)^{-\frac{1}{3}} \\ \rho &= F_+(x,y)^{\frac{1}{6}} F_-(x,y)^{\frac{1}{6}} \end{aligned} \tag{3.17}$$

Note that the metric factor for the base space Σ becomes non-trivial for the deformed solution. Another interesting feature of this solution is that the size of each three sphere at the part of the boundary where it does not vanish, i.e. $y = 0$ for f_2 and $y = \pi/2$ for f_3, becomes x dependent in contrast to the $AdS_4 \times S^7$ solution where it is constant.

The AST fields along the two three spheres are non-vanishing for the deformed solution. The formulae for the fieldstrengths (3.9) can be integrated and one obtains the following expressions for the AST potentials.

$$\begin{aligned} b_2 &= -\frac{8\lambda\sqrt{1+\lambda^2}\, \sin^4(y)}{\mathrm{ch}^2(2x)\; F_-(x,y)} \\ b_3 &= \frac{8\lambda\sqrt{1+\lambda^2}\, \cos^4(y)}{\mathrm{ch}^2(2x)\; F_+(x,y)} \end{aligned} \tag{3.18}$$

The explicit formulae for the AST potential b_1 are also easy to calculate but will not be needed in the following. We have checked that the solutions indeed satisfies all equations of motion of 11-dimensional supergravity. The solution preserves sixteen of the thirty two supersymmetries by construction [23].

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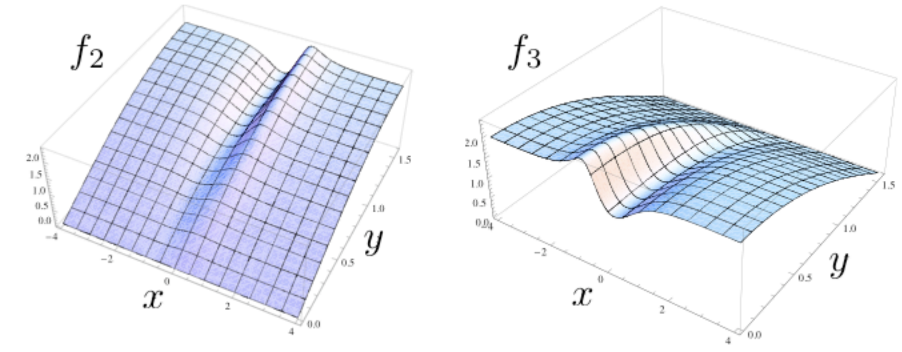

Figure 1. Metric factors of the two spheres for the deformation $\lambda = 1$.

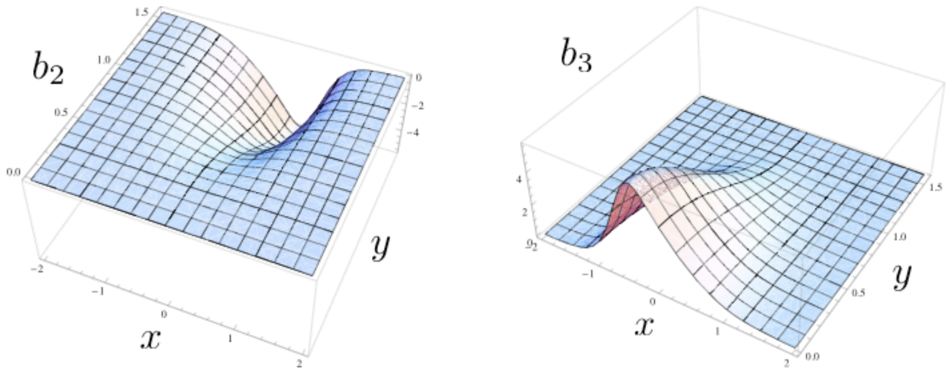

Figure 2. AST potentials along the two three spheres for the deformation $\lambda = 1$.

3.4 Holographic interpretation

In the previous section we utilized the coordinate system in which Σ is a strip parametrized as $\Sigma = \{(x, y), x \in \mathbf{R},\ 0 \leq y \leq \pi/2\}$. The boundaries of Σ at $y = 0, \pi/2$ are the location where the volume of the first and second three sphere vanishes. For fixed x the finite y-interval together with the two three spheres produces a deformed seven sphere with $\mathrm{SO}(4) \times \mathrm{SO}(4)$ symmetry. The asymptotic AdS region is reached by taking $x \to \pm\infty$. In this limit the leading terms of the metric factors (3.17) behave as follows,

$$ds^2 = \frac{1}{4} e^{\pm 4x}\, \frac{-dt^2 + ds^2 + dz^2}{z^2} + 4\sin^2(y)\ ds^2_{S_2^3} + 4\cos^2(y)\ ds^2_{S_3^3} + 4(dx^2 + dy^2) + o(e^{\mp 4x}) \tag{3.19}$$

which is the asymptotic form of $AdS_4 \times S^7$. The leading terms for the 3-form potential C_3 in the limit $x \to \pm\infty$ can be obtained from (3.18),

$$C_3 = \frac{1}{16} e^{\pm 6x} \hat{\omega}_{AdS_3} - \frac{32\lambda}{\sqrt{1+\lambda^2}} e^{\mp 4x} \Big(\sin^4(y)\ \hat{\omega}_{S_2^3} - \cos^4(y)\ \hat{\omega}_{S_3^3} \Big) + o(e^{\mp 2x}) \tag{3.20}$$

The first term in (3.20) produces the four form flux supporting the $AdS_4 \times S^7$ solution. The second term vanishes when $\lambda = 0$, which corresponds to the undeformed solution. Note that the deformed solution has a nontrivial profile for the AST potential along the spheres, but it is easy to see that the conserved M5-brane charge integrates to zero.

To make contact with the discussion of section 2, we first note that the dependence on the y and sphere coordinates in the second and third terms in (3.20) corresponds to a specific AST spherical harmonic on the sphere. The Kaluza-Klein reduction [20] produces the pseudo-scalar field ϕ of mass $m^2 = -2$ transforming in the $\mathbf{35_c}$ of the SO(8) R-symmetry.

We can relate the coordinate x which parametrizes the AdS_3 slicing coordinate in the strip parametrization to the coordinate μ which was employed in section 2 by,

$$\mu \mp \pi/2 = e^{\mp 2x} \tag{3.21}$$

valid in the limit $x \to \pm\infty$. It follows from (3.20) that the pseudo-scalar field mode associated with the KK reduction of the AST potential has the following behavior,

$$\lim_{\mu \to \pm\pi/2} \phi = \text{constant} \times (\mu \mp \pi/2)^2 + o(\mu \mp \pi/2)^4 \tag{3.22}$$

Consequently, the holographic behavior of ϕ is exactly of the type discussed in section 2. The holographic interpretation of our solution is that a dimension two operator which preserved SO(4) $\times$ SO(4) R-symmetry, as well as sixteen supersymmetries, is sourced at a 1+1-dimensional interface/defect.

In the context of BPS Janus solutions in Type IIB the related CFT analysis of supersymmetric interface/defects in $\mathcal{N} = 4$ super Yang Mills was carried out in [10, 11, 24, 25]. It would be interesting to pursue a similar analysis for the M2-brane CFT, where significant progress in the formulation of the theory has been made recently [26–28]. See [29, 30] for other attempts to obtain Janus like solutions for the M2-brane worldvolume theory.

4 Janus solutions in ABJM theory

Recently, ABJM [12] found a new AdS/CFT correspondence between certain quotients of $AdS_4 \times S^7$ in M-theory, and 2+1-dimensional CFTs which preserve $\mathcal{N} = 6$ superconformal symmetry. The remarkable benefit of this correspondence lies in the fact that it provides a standard field theory description of the corresponding 2+1-dimensional CFTs. In this section, we show that for each of the ABJM quotients, there exists a corresponding regular ABJM Janus solution, which may be obtained by quotienting the M-theory Janus solution of the previous section à la ABJM.

4.1 Construction of the ABJM Janus solution

The supergravity description of ABJM theory is given by the quotient $AdS_4 \times S^7/Z_k$ of the maximally symmetric vacuum $AdS_4 \times S^7$ by the cyclic group Z_k for $k \geq 1$. Here, Z_k acts on S^7, but does not act on AdS_4. The action of Z_k on S^7 has no fixed points, and the resulting quotient may be viewed as a line bundle over CP_3 whose fiber S^1 has radius $1/k$. The quotient breaks the SO(8) R-symmetry to SU(4) $\times$ U(1).

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The action on S^7 may be rendered more explicit by embedding S^7 into $\mathbf{R}^8$, and parametrizing $\mathbf{R}^8$ by four complex coordinates z_i, with $i = 1, 2, 3, 4$. The Z_k transformation is then defined by $z_i \to z_i \, e^{2\pi i/k}$ for any integer $k \geq 1$, and the quotient is obtained by identifying the points z_i and $z_i \, e^{2\pi i/k}$ for all four directions $i = 1, 2, 3, 4$.

To construct the ABJM Janus solution, it will be more useful to exhibit the Z_k transformations in terms of real coordinates, on which the action of the full SO(8) is manifest. The corresponding representation matrix γ in the group SO(8) is then given by $\gamma = e^{2\pi H/k}$, where H is the following generator of the Lie algebra SO(8),

$$H = \begin{pmatrix} H_2 & 0 \\ 0 & H_3 \end{pmatrix} \qquad\qquad H_2 = H_3 = \begin{pmatrix} \varepsilon & 0 \\ 0 & \varepsilon \end{pmatrix} \qquad\qquad \varepsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{4.1}$$

The notation H_2 and H_3 has been introduced for the following reason. The M-theory Janus solution is characterized by the breaking of the Lie algebra SO(8) to $\mathrm{SO}(4)_2 \oplus \mathrm{SO}(4)_3$, the two $\mathrm{SO}(4)_i$ subalgebras being the isometries of the spheres S^3_i for $i = 2, 3$ of the Janus solution in (3.1). In the above partition of H into the direct sum of H_2 and H_3, the generators are arranged so that $H_2 \in \mathrm{SO}(4)_2$, and $H_3 \in \mathrm{SO}(4)_3$. As a result, the action of Z_k on S^7 descends to the Janus solution as a Z_k action on S^3_2 and S^3_3, whose explicit form is given by the SO(4) Lie group matrices $\gamma_2 = e^{2\pi H_2/k}$ and $\gamma_3 = e^{2\pi H_3/k}$.

The action of $\gamma_{2,3}$ on $S^3_{2,3}$ is again without fixed points. Furthermore, the action of $\gamma_{2,3}$ leaves all the other ingredients of the Ansatz of (3.1), (3.2), (3.3), and of the solution functions h and G invariant. Thus, we are guaranteed that the quotient of the M-theory Janus solution by the action of Z_k will produce a regular family of solutions, parametrized by the same parameter λ as the M-theory Janus was. The only change to the geometry resides in the quotient of $S^3_2 \times S^3_3$ by Z_k.

The quotienting of S^3 by Z_k reduces the isometry group from SO(4) to an $\mathrm{SU}(2) \times \mathrm{U}(1)$ subgroup. Thus, we expect the ABJM Janus solution to have a compact bosonic symmetry group $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)^2$, as well as, of course, the full isometry SO(2, 2) of AdS_3, producing a total bosonic symmetry group

$$\mathrm{SO}(2,2) \times \mathrm{SO}(4) \times \mathrm{U}(1)^2 \tag{4.2}$$

Supersymmetry is also reduced under quotienting by Z_k, $k \neq 1, 2$. This reduction is entirely due to the reduction of the number of Killing spinors on $S^3_2 \times S^3_3$, and proceeds in parallel to the corresponding reduction on S^7. The 4 independent Killing spinors on $S^3_2 \times S^3_3$ are reduced to only 3 Killing spinors on $(S^3_2 \times S^3_3)/Z_k$. For $AdS_4 \times S^7$, this effect reduces the total number of supersymmetries from 32 to 24, while for the M-theory Janus solution, it reduces the number of supersymmetries from 16 to 12. Given these bosonic and supersymmetries, the corresponding invariance superalgebra of the ABJM Janus solution is readily obtained,

$$\mathrm{OSp}(3|2, \mathbf{R}) \times \mathrm{OSp}(3|2, \mathbf{R}) \tag{4.3}$$

which is a subgroup of the $\mathrm{OSp}(4|2, \mathbf{R}) \times \mathrm{OSp}(4|2, \mathbf{R})$ algebra of the M-theory Janus solution.

4.2 Structure of the supergravity multiplets

As pointed out in [31, 32], the spectrum of the KK reduced theory can be obtained by decomposing the KK spectrum of $AdS_4 \times S^7$ with respect to SU(4) $\times$ U(1) and keeping only the zero charge sector of the U(1). For the fields of $N = 8$ gauged supergravity in table 2, this implies the following decompositions of SO(8) representations under its SU(4) $\times$ U(1) subgroup,

$$\begin{aligned} \mathbf{28_v} &\to \mathbf{1_0} + \mathbf{6_2} + \mathbf{6_{-2}} + \mathbf{15_0} \\ \mathbf{35_v} &\to \mathbf{10_2} + \overline{\mathbf{10}}_{\mathbf{-2}} + \mathbf{15_0} \\ \mathbf{35_c} &\to \mathbf{10_2} + \overline{\mathbf{10}}_{\mathbf{-2}} + \mathbf{15_0} \end{aligned} \tag{4.4}$$

As a result, out of the 35 pseudo-scalar 0^- fields transforming in the $\mathbf{35_c}$ of SO(8) in the KK spectrum, fifteen survive the quotient and transform in the **15** of SU(4). These fields have mass $m^2 = -2$ and are dual to dimension 2 operators. The linearized analysis of section one is expected to apply to these states, and it provides further evidence for the existence of Janus-like interface/defect solutions in this theory.

It is possible to characterize the dual operator using the ABJM worldvolume theory. The scalars and fermion fields transform as bi-fundamentals of a U(N) $\times$ U(N) gauge theory. There is an SU(2) doublet of bosons A_1, A_2 and fermions λ_1, λ_2 which transform as $(N, \bar{N})$ under U(N) $\times$ U(N) gauge symmetry, whereas a second pair of bosons B_1, B_2 and fermions χ_1, χ_2 transforms as $(\bar{N}, N)$. The boson and fermion fields can be assembled into the following multiplets [12, 33, 34],

$$Y^A = (A_1, A_2, B_1^\dagger, B_2^\dagger) \qquad\qquad \Psi^A = (\lambda_1, \lambda_2, \chi_1^\dagger, \chi_2^\dagger) \tag{4.5}$$

which transform in the **4** representation of SU(4), wheras the conjugate fields $(Y^A)^\dagger = Y_A$ and $(\Psi^A)^\dagger = \Psi_A$ transform in the $\bar{\mathbf{4}}$ of SU(4). The following operators

$$O_1 = \mathrm{tr}\left(Y^A Y_B - \frac{1}{4}\delta^A_B Y^C Y_C\right) \qquad\qquad O_2 = \mathrm{tr}\left(\Psi^A \Psi_B - \frac{1}{4}\delta^A_B \Psi^C \Psi_C\right) \tag{4.6}$$

are conformal primary operator of dimensions $\Delta = 1$ and $\Delta = 2$ respectively which transform as the **15** of SU(4). Therefore, they can be identified with the surviving scalar and pseudo-scalars in the quotient (4.4). Applying the linearized analysis of section 2 to the field dual to one O_2 one expects that a Janus-like interface/defect solution exists for the $AdS_4 \times S^7$ quotient.

5 Discussion

In this paper we have presented exact solutions of 11-dimensional supergravity which are holographically dual to inserting a dimension 2 operator along a 1+1-dimensional interface/defect in the maximally supersymmetric 2+1-dimensional CFT. The M-theory Janus solution preserves 16 supersymmetries and has SO(2, 2) $\times$ SO(4) $\times$ SO(4) isometry group, while the ABJM Janus solution preserves 12 supersymmetries and has SO(2, 2) $\times$ SO(4) $\times$

$U(1)^2$ isometry group. The symmetries combine into an $OSp(4|2, R) \otimes OSp(4|2, R)$ invariance superalgebra of the M-theory Janus solution, and an $OSp(3|2, R) \otimes OSp(3|2, R)$ invariance superalgebra for the ABJM Janus solution. Both are subgroups of the supergroup $OSp(8|4, R)$ of the $AdS_4 \times S^7$ vacuum [35]. These solutions are analogs in M-theory of the Janus solution of Type IIB, even though no dilaton is present in M-theory.

There are several interesting open questions and directions for further research.

1. Can the 11-dimensional Janus solutions be expressed as solutions solely of the massless multiplet of the $\mathcal{N} = 8$ gauged supergravity.

2. Exact solutions, such as the M-theory and ABJM Janus solutions obtained here, may be used to calculate interesting quantities using the machinery of AdS/CFT. For example, application of the methods developed in [10, 36, 37] could be used to calculate correlation functions in the presence of the interface/defect.

3. As the ABJM theory enjoys a well-understood field theoretic CFT description, one may classify the possible interface/defect terms along the lines of [11] and [24], establish their symmetries, and derive the associated supergravity solutions.

4. In Type IIB theory, the simplest Janus solution breaks all supersymmetries, has a non-trivial dilaton profile, and vanishing 3-form flux fields. A natural question is whether M-theory Janus solutions exist with no, or further reduced supersymmetry, and whether the corresponding supergravity solutions lend themselves to exact construction.

5. Finally, does Type IIB supergravity support Janus-type solutions whose dilaton is constant, but whose 3-form and 5-form fields vary spatially ? The corresponding CFT dual would be $\mathcal{N} = 4$ super-Yang-Mills with identical gauge couplings on both sides of the interface/defect, and dimension 3 operators localized on this defect/interface. In other words, does Type IIB admit the Janus-type solutions characteristic of M-theory?

We plan to address some of these questions in the near future.

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